# Flow Characteristics in Horizontal Fluidized Solids Transport

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**The flow characteristics of dense solid-gas mixtures transported through horizontal pipes were studied with glass beads and coal powders of various sizes (0.0028 to 0.0297 in.) in 1/2-, 3/4- and 1-in. glass pipes and a 1/4-in. steel pipe. Fluidized-bed feeders were utilized, thus permitting solid-gas ratios considerably higher (range 80 to 750) than those possible with conventional pneumatic transport. When such high solid-gas ratios are used, the flow of mixtures in transport lines is characterized by a large amount of slippage between gas and solids. The flow pattern is discussed on the basis of visual observation through glass pipes.**

**A simple and interesting velocity relationship was noted, namely that the average gas velocity is about twice as large as the average solid-particle velocity. The solid-particle velocities and solid loadings in the pipe line were found to be primary factors affecting pressure drops, and the particle sizes and shapes, on the other hand, exerted a very slight effect on the pressure drops. This is apparently due to the fact that the solids move predominantly in the bottom of the pipes as agglomerated masses rather than as individually suspended particles. A pressure-drop correlation for the dense solid-gas mixtures is proposed, and applications and limitations of the correlation are shown.**

Although pneumatic conveying of granular solids has been practiced for many years, the use of fluidized-bed feeders is a relatively recent development. Characterized by high solid-gas ratio mixtures at comparatively high pipe-line capacities (*1*, *2*, *11*), this use permits an operation at much lower cost than conventional pneumatic conveying (*11*). The technique has found many interesting applications, and like many other chemical engineering developments it is in extensive use even though its exact nature—in this case the nature of the flow—is not completely understood. Recent studies have recognized the importance of particle velocity, density, and rheological behavior of solid-gas systems; however the difficulty of accurately measuring these quantities has precluded the establishment of a clear picture of the mutual relations in solid-gas flow phenomena. It is the purpose of the present investigation to study the flow patterns of dense solid-gas systems, particularly the interrelationship of solid and gas velocities, solid-gas ratio, and pressure drop in horizontal transport lines.

In earlier works the study of solid-gas flow was limited to the pneumatic conveying of a few specific materials of relatively large particle size. Gästerstadt (*7*) found that the specific pressure drop was related to the solid-gas ratio, and Cramp and Priestley (*6*) developed an empirical relationship for determining the pressure drop for pneumatic conveying in the vertical duct. Wood and Bailey (16) reported that at high solid-gas ratios the system became nonhomogeneous, with greatly differing flow characteristics from the lean solid-gas flow. The pressure-drop data for both horizontal and vertical pneumatic transport lines were correlated by Vogt and White (*15*) by dimensionless groups. Belden and Kassel (*3*) expressed total pressure drop as the sum of the static and friction terms which had previously been combined into one term by Vogt and White. Their findings indicate that the pressure drop is nearly independent of tube-diameter–particle-diameter ratio. Hariu and Molstad (*8*) were able to report average particle velocity, slip velocity, and solids static pressure by means of a dispersed-solids-density calculation. Particle velocities have been studied in lean phase flow by Khudyakov and Chukhanov (*10*), Uspenskii (*14*), Clark, *et al.* (*5*), Mehta (*12*), and Hinkle (*9*). However most of these studies are not directly applicable to dense-phase transport. Since the particles in dense solid-gas systems tend to precipitate and move along the bottom of the pipes, it is apparent that the flow pattern must be greatly different from pneumatic transport, in which most of the particles are carried in suspension. Using fluidized-bed feeders, Albright, *et al.* (*1*, *2*), measured pressure drops for powdered-coal systems at comparatively high solid loadings. Koble, *et al.* (*11*), and Carney (*4*) reported pressure-drop data for dense-phase solids transport as well. Their correlations were however tentative, and no generalization for prediction of the pressure-drop or solid-delivery rate resulted.

## EXPERIMENTAL

Figure 1 is a diagram of the experimental unit. Solids charged to the hopper could be added continuously to the fluidizer by means of valve 3. When a suitable quantity of solids had been charged into the column (until a bed height of approximately 2.5 ft. was attained), air was introduced through the conical adaptor to the bottom of the fluidizer column, and the solid particles were fluidized. The plug valves (4) were opened, and the solid particles allowed to flow through the pipe. With steady state conditions attained, the solids were collected in the receiver (2) and the time was noted for collection. The carrier air, separated from the solids at the receiver (2), was measured by a dry test meter. The pressure drop for transport across the pipe section was recorded by the Foxboro type of differential pressure recorder. At the end of the run the two plug valves (4), one at the entrance and the other at the exit end of the pipe, were closed simultaneously, the solids holdup in the pipe being discharged by the blowing of air through the test section, and the collected solids were weighed. The fluidized-solids level in the fluidizer was maintained as constant as possible during the operation by the continuous addition of solids from the hopper. Preliminary tests indicated that the solids flow rate remained constant within the limits of experimental error throughout the entire period of the run. The solids flow rate as well as solid-gas ratio were varied by changing the pressure in the fluidizer and the fluidizing air rate. Since the pressure drops in such dense solids-gas flow fluctuated, in some cases considerably, the integrated average pressure drops were evaluated by a planimeter and the charts from the differential-pressure recorder for the entire period of the run. Flow data* were collected for a total of approximately 200 runs with glass beads of 0.011-, 0.0058- and 0.0028-in. diameter and coal powder of 0.0297-, 0.0197-, and 0.0044-in. diameter, flowing through 0.5-, 0.75-, and 1.0-in. I.D. glass pipes and 0.364-in. I.D. steel pipe. The glass beads were essentially spherical. The average diameter of the coal particles was determined by screen analysis; the solid particles were periodically inspected under the microscope, and no appreciable attrition was observed. The densities of the glass beads and coal powders were 156 and 81 lb./cu. ft. respectively.

## MODE OF FLOW

Visual observation of the behavior of solids motion in a glass pipe indicated that there may be four modes of flow by which the solid particles travel. Since the solid particles are quite uniformly dispersed throughout the fluidized bed before they are introduced into the pipe, the particles in the early section of the transport line, immediately adjacent to the fluidizer, appear to be evenly distributed (Figure 2*a*). The particles are accelerated to their terminal velocities in this section and thereafter will tend to settle in the bottom portion of the pipe, forming dunes as shown in Figure 2*b*. Because of particle settling, solids holdup in this section of the pipe will be larger

*Tabular material has been deposited as document 5913 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$2.50 for photoprints or \$1.75 for 35-mm. microfilm.

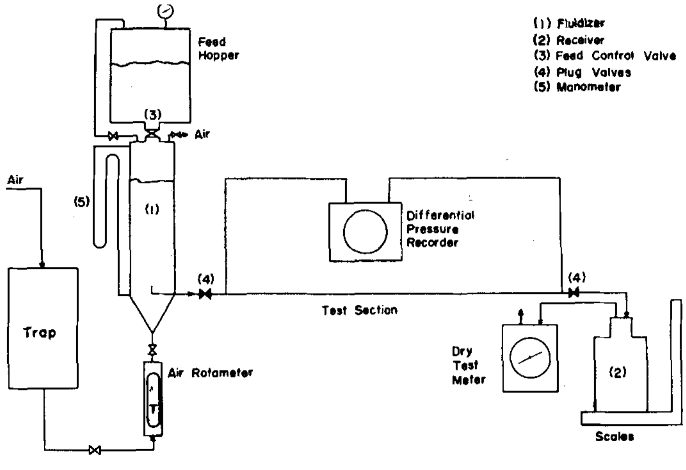


Fig. 1. Schematic diagram of equipment.

than in the early section, and consequently the average particle velocity in such moving dunes is less than when the particles are in true suspension. Solids flow takes place by way of moving from one dune to the next, undergoing deceleration and acceleration alternatively. The distance between the dunes, which depends on the solids flow rate as well as gas flow rate, varies from several inches to several feet. In high velocity flow the dunes either do not form or appear only at the exit end of the pipe. For most runs made in 1-in. glass pipe, the length of the pipe through which no precipitation of the particles occurred was approximately 3 ft. In low-velocity flow the dunes are formed more readily, and length and height of the dunes are correspondingly larger. As the solid loadings increase, the dunes grow, sometimes almost to the same height of the pipe diameter, and small ripples will appear traveling along the top of the thick solid layer, the greater portion of which is practically stationary. This is shown in Figure 2*d*. Further increase of solid loadings will eventually result in complete blockage of the transport line.

Sometimes, depending on the nature of solids and solid-gas ratio, intermittent flow of gas and solids in alternate slugs will occur, rather than dune formation (Figure 2*c*). The pressure drops over the transport line are then unstable, a condition which is reflected in considerable fluctuation on the manometer.

Fig. 2a. High velocity flow or near inlet section.

Fig. 2*b*. Segration of particles and dunes formation.

Fig. 2*c*. Intermittent flow of gas and solids in alternate slugs.

Fig. 2*d*. Low velocity flow; solid ripples travel along the top of stationary solid layer.

## AVERAGE SOLID AND GAS VELOCITY

Under such irregularities in flow, particle velocities must be expected to vary considerably across the pipe cross section. Moreover the velocities also change from one point to the next along the pipe, even after the particles have left the supposed acceleration region. Consequently it is only logical to express the particle velocity as the average velocity of all the particles for the entire length of transport line under consideration. From the equation of continuity

$$G_s = \rho_{ds} U_s \tag{1}$$

where the dispersed solid density may be calculated from the experimental measurement of the solid holdup in the transport lines. If $W_s$ is the weight of solids retained in the pipe line of length $L$, then $\rho_{ds} = W_s/(\pi D_t^2/4)L$. If the solid mass velocity is known, the average particle velocity can be evaluated from Equation (1).

The relationships between the dispersed solid density and the apparent over-all solid-gas mixture density for coal particles and glass beads are shown in Figures 3 and 4. The apparent over-all solid-gas-mixture density was obtained by dividing the weight of solid collected by the total volume of solid-gas flow. Figures 3 and 4 indicate considerable slippage between gas and solid in the dense solid-gas flow. Had there been very little or no slippage between the gas and the solids, $\rho_{ds}$ would be nearly equal to $W_s/V$. For coal particles as well as glass beads

$$\rho_{ds} = k(W_s/V) \tag{2}$$

where $k$, varies between two and three, corresponding to particle diameters of 0.0044 to 0.0297 in. an indication that $\rho_{ds}$ depends only slightly on particle diameter. The effect of solids mass velocity on particle velocity is shown in Figures 5 and 6. By means of a cross plot at a constant particle velocity the solids mass velocity can be shown to vary with the minus 0.7 power of the pipe diameter. Based on this relation, an empirical correlation between particle velocity and solid particle densities is presented in Figure 7. While it has been demonstrated that the particle velocity depends on solid mass velocity, pipe diameter of transport line, and the solid particle density, no consistent effect of solid-gas ratio has been found. Since the correlation is not dimensionless, it must be borne in mind that not all the effects of either solid or fluid upon particle velocity have been considered; however the correlation presented in Figure 7 serves to predict the approximate particle velocity of which direct measurement is not practicable.

The average linear gas velocity in transport line can be found similarly from the continuity equation for the gas,

$$G_a = \rho_{da} U_a \tag{3}$$

The dispersed gas density $\rho_{da}$ is obtained from $\rho_{da} = W_a/(\pi D_t^2/4)L = (1 - \rho_{ds}/\rho_s)\rho_a$. Previous workers evaluated the gas velocity based on the empty pipe, but this is an approximation which holds only where the volume occupied by the solid in the lines is negligible. In the dense solid-gas flow system the solids occupy a large portion of the total flow space; the linear gas velocity therefore is

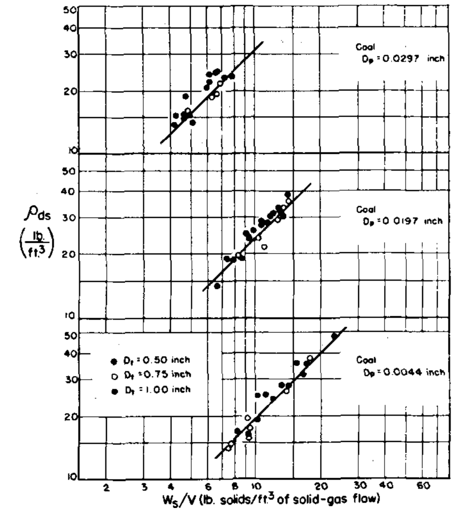


Fig. 3. Dispersed solid density and apparent over-all solid-gas mixture density.

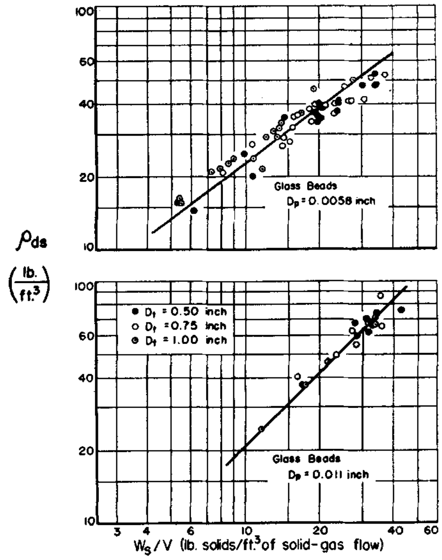


**Fig. 4. Dispersed solid density and apparent over-all solid-gas mixture density.**

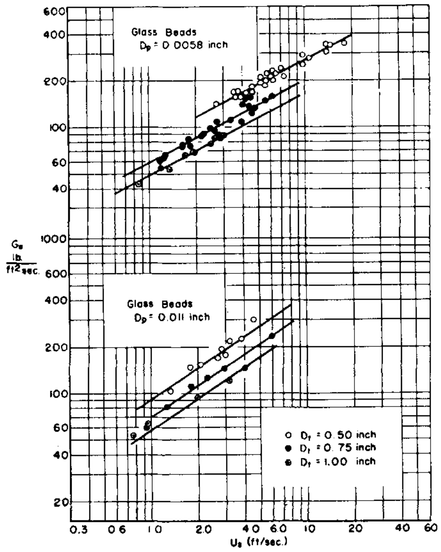


**Fig. 6. Solids mass velocity and particle velocity.**

much greater than the superficial gas velocity.

The extent of slippage between gas and solid is shown in Figure 8, in which the solid velocity is plotted against the gas velocity. An arithmetic average of entrance and exit gas velocity was used in the plot, owing to the expansion of the gas in the transport line. Figure 8 indicates the gas velocity to be approximately twice as large as the solid velocity independent of particle sizes. Thus

$$U_a \cong 2U_s \tag{4}$$

The data of Hariu and Molstad (*8*) in vertical transport, which resemble the dense solid-gas flow, are also included for comparison. It is interesting to note that

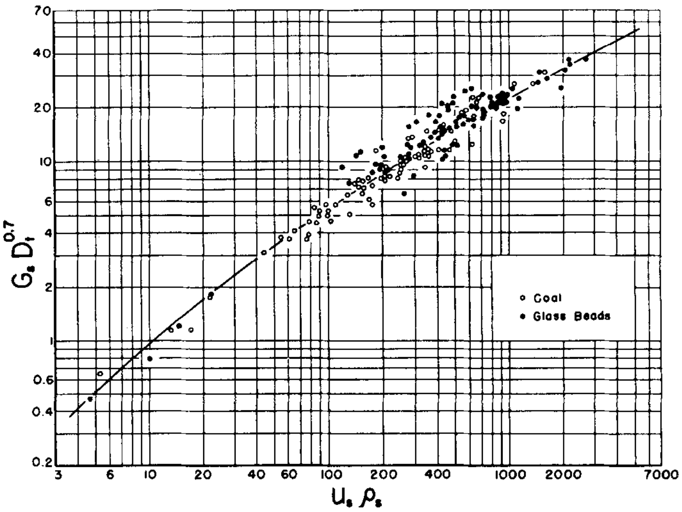


**Fig. 7. Solids mass velocity and particle velocity.**

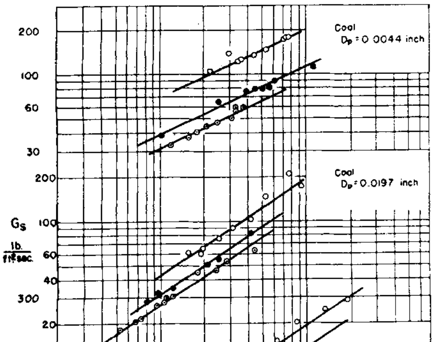


**Fig. 5. Solids mass velocity and particle velocity**

such a simple relationship exists between velocity of gas and solid in seemingly a very complex system. Since it is difficult to obtain accurate velocity data, the trend indicated appears to be significant.

## PRESSURE DROP

For horizontal transport the total pressure drop may be expressed as

$$\Delta P_t = \Delta P_{aa} + \Delta P_{as} + \Delta P_d + \Delta P_f \tag{5}$$

When the solid particles reach the terminal velocity

$$\Delta P_t = \Delta P_d + \Delta P_f \tag{6}$$

$\Delta P_d$ may be evaluated by the equation

$$\Delta P_d = G_a \, \Delta U / g \tag{7}$$

and is related to drag coefficient according to

$$\Delta P_d = \frac{C_d A_b \rho_g \, \Delta U^2}{2g(\pi \, D_t^2/4)} \tag{8}$$

Using the modification of the Fanning friction factor, one can express $\Delta P_f$ by

$$\Delta P_f = \frac{2fU_s^2 \rho_{ds} L}{g \, D_t} \tag{9}$$

Since no significant difference in pressure drop was observed between the first and second 10-ft. test sections of the pipe, the solid particles were considered to have left the initial acceleration region. The values of $\Delta P_f$ and $\Delta P_d$ are calculated. The results indicate that for a 10-ft. section $\Delta P_d$ is in no case larger than 1.0% of the total pressure drop. If the transport line is long, therefore, the total pressure drop in dense solid-gas mixture can be estimated as equal to $\Delta P_f$, the friction of solid particles against particles themselves and against the gas.

The pressure drop is correlated empirically by two dimensionless groups to the particle velocity (Figure 9). The trend can be represented by

$$\left(\frac{\Delta P_t}{L\rho_{ds}}\right)\left(\frac{D_t}{D_p}\right)^{1/4} = 2.5U_s^{0.45} \tag{10}$$

The effect of particle diameter or shape on the pressure drop is again proved to be very small in comparison with the solid dispersed density or solid loadings as well as particle velocity in the transport line. As previously pointed out, this is due to the fact that the solid particles travel predominantly in the bottom of the pipes as agglomerated masses, and thus the effect of diameter and shape practically vanishes. The effect of pipe diameter is not apparent from the equation, since $\rho_{ds}$ is also the function of pipe diameter. Mitlin (*13*) also reported that the pressure drop is practically independent of particle size and is governed chiefly by weight of material in the line.

Since air was the only fluid employed, the correlation is not dimensionless, and it should be noted that not all the effects on the pressure drop are taken into consideration in Equation (10). With Equation (4) used, the solid-gas–ratio data reported by Koble (*11*) and Carney (*4*) for the ratios above 80 were recalculated and are shown on the same plot (Figure 9) for comparison. Both investigators employed a ¼-in. pipe. Their results seem to agree within experimental error with the proposed correlation.

The deviation between the values reported by previous investigators is due probably to the method of pressure-drop measurement in the transport line. While some measured the static pressure difference between the fluidizer and the receiver, others determined the pressure drops through taps directly attached to the line. In addition to the inaccuracies caused by the entrance and exit losses in the former method of pressure-drop measurement, both methods may possibly be inflicted with inconsistencies due to particle acceleration. Hinkle (*9*) reported that pressure drop due to particle acceleration was a significant fraction of the total pressure drop in the pneumatic conveying system. However in dense solid-gas flow of the type considered, the length of pipe necessary to reach terminal particle velocity, owing to the considerably large frictions exerted between the particles as well as with pipe walls, is probably much less than that expected in lean solid-gas flow, such as in pneumatic conveying. In fact, owing to the segregation of the solid particles in the bottom of the pipe at the downstream end of the transport line, the particles are subjected to repeated positive and negative acceleration, and therefore it is very difficult to determine over what length of pipe the particles are being accelerated or decelerated. Another source of error is the considerable static electricity generated while the solid particles are passed through pipes. The dryness of the carrier gas seems to affect this considerably, and proper grounding is necessary to minimize static effects, especially when a very dry gas and solid particles having poor electrical conductivity are used.

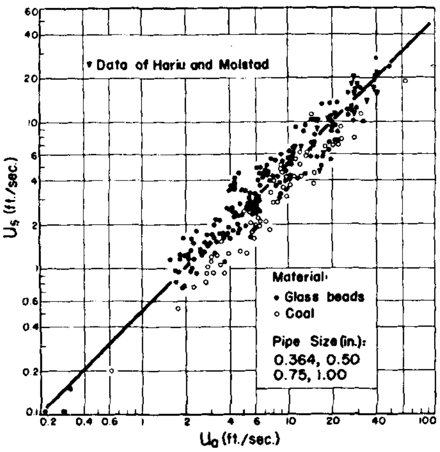


**Fig. 8. Solid particle velocity vs. gas linear velocity.**

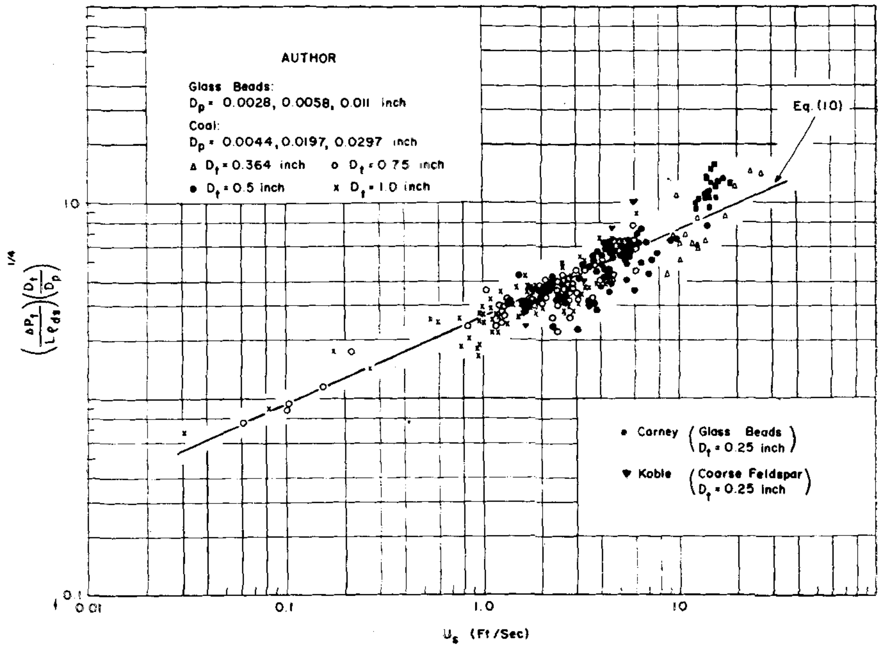


**Fig. 9. Pressure-drop correlation.**

## APPLICATION

The following illustrates the application of the developed correlation.

### Problem

Coal powder ($D_p$ = 0.005 in. and $\rho_s$ = 81 lb./cu. ft.) is to be transported from a fluidized bed to a gasification reactor through a horizontal pipe of 1-in. I.D. and 10-ft. length for manufacturing synthesis gas. For a solid flow rate of 785 lb./hr., estimate the amount of air required to transport the solid, the line pressure drop, the average solid particle velocity, and the average slip velocity.

### Solution

The average solid-particle velocity can be obtained from Figure 7.

Since $G_s = 785 \times 144 \times 4/(3600 \times 3.14) = 40$ lb./sq. ft.-sec.

$G_s D_t^{0.7} = 40 \times 0.176 = 7.04$

$\rho_s U_s = 150$ lb./cu. ft.

$U_s = 150/81 = 1.85$ ft./sec.

$U_a = 2 \times 1.85 = 3.70$ ft./sec.

The average slip velocity is therefore

$\Delta U = U_a - U_s = 1.85$ ft./sec.

$\rho_{ds} = G_s/U_s = 40/1.85 = 21.6$ lb./cu. ft.

$\rho_{da} = (1 - \rho_{ds}/\rho_s)\rho_a = (1 - 21.6/81)\ 0.076 = 0.0557$ lb./cu. ft.

The gas mass velocity is

$G_a = \rho_{da} U_a = 0.0557 \times 3.70 = 0.206$ lb./sq. ft.-sec. or 4.04 lb./hr.

The solid-gas ratio is

$R = G_s/G_a = 40/0.206 = 194$ lb. solid/lb. air

The pressure drop is evaluated from Equation (10):

$\Delta P_t = 2.5 \times (1.85)^{0.45} \times 10 \times 21.6 \times (0.005)^{0.25}$
$= 189$ lb./sq. ft./10 ft.

## CONCLUSION

The flow characteristics of dense solid-gas mixtures, introduced from a fluidized bed feeder into horizontal transfer lines, differ considerably from flow of lean solid-gas suspensions. It was observed that the solid particles tend to segregate in the bottom sector of the conveying pipe and travel as agglomerated masses. Particle size and shape seem to have only a very slight effect on flow characteristics. Large slippages between gas and solid particles in transport line were observed, and a simple relationship that the average gas velocity is about twice as large as the average solid-particle velocity was noted. Empirical correlations for predicting the solid-particle velocity and pressure drop in horizontal line are proposed. The length of pipe required for the solid particles to attain the terminal velocity is difficult to evaluate correctly, since the solids are subjected to repeated positive and negative acceleration.

Although several points have been clarified by the results of the present investigation, the understanding of the problem of dense solid-gas transport is far from complete because of the complexity of the solids flow pattern. Hence much additional work is required before a conclusive generalization is in sight.

### NOTATION

$A_h$ = cross-sectional area of agglomerated mass of flow in pipe ($L^2$)
$C_d$ = drag coefficient (no dimension)
$D_p$ = average particle diameter ($L$)
$D_t$ = pipe diameter ($L$)
$f$ = friction factor (no dimension)
$g$ = gravitational acceleration ($L\theta^{-2}$)
$G_a$ = gas mass velocity in pipes ($ML^{-2}\theta^{-1}$)
$G_s$ = solid mass velocity in pipes ($ML^{-2}\theta^{-1}$)
$k$ = constant for a given particle size
$L$ = length of the transport line ($L$)
$R$ = solids to gas ratio (no dimension)
$U_a$ = average linear velocity of gas ($L\theta^{-1}$)
$U_s$ = average linear velocity of solid particles ($L\theta^{-1}$)
$V$ = total volume of solid and gas collected in the receiver ($L^3$)
$W_a$ = weight of gas ($M$)
$W_s$ = weight of solid ($M$)

#### Greek Letters

$\Delta P_{aa}$ = pressure drop due to acceleration of gas ($FL^{-2}$)
$\Delta P_{as}$ = pressure drop due to acceleration of solid particles ($FL^{-2}$)
$\Delta P_d$ = pressure drop due to body drag ($FL^{-2}$)
$\Delta P_f$ = pressure drop due to friction ($FL^{-2}$)
$\Delta P_t$ = total pressure drop ($FL^{-2}$)
$\Delta U$ = slip velocity between gas and solid particles ($L\theta^{-1}$)
$\rho_a$ = density of gas ($ML^{-3}$)
$\rho_{da}$ = dispersed gas density ($ML^{-3}$)
$\rho_{ds}$ = dispersed solid density ($ML^{-3}$)
$\rho_s$ = solid particle density ($ML^{-3}$)

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*Manuscript received September 5, 1956; revision received November 17, 1958; paper accepted November 17, 1958. Paper presented at A.I.Ch.E. Montreal meeting.*


# Effective Diffusivity of Packed Bed


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**From time to time, A.I.Ch.E. Journal presents translations of certain technical articles written by our Japanese colleagues in their own language. These translations are made by Kenzi Etani, who received his B.S. in chemical engineering in 1953 at the Tokyo Institute of Technology and his M.S. in 1955 at M.I.T. He is associated with Stone & Webster and is an associate member of American Institute of Chemical Engineers. He is also a member of the Society of Chemical Engineers, Japan, and the Japan Oil Chemists' Society. His offer to help break down the language barrier is acknowledged.**

**Abstracts, notation, literature cited, tables, and figure captions not published here appear in English in the original paper. No figures will be reproduced in these translations.**

**The following article was published in *Chemical Engineering (Japan)*, 21, pages 17–25 (1957).**

The purpose of this experiment was to obtain the relation between effective diffusivity and fractional voids under nonflowing conditions. The results were compared with electric conductivity.

### APPARATUS AND EXPERIMENT

The apparatus used in this experiment is shown in Figure 1. The diameters of the tubes were 3.50 and 3.05 cm. The height of the packed bed was changed from 5 to 15 cm. All apparatus was kept at the constant temperature of 40 ± 0.2°C. in a constant-temperature bath. Air having the same temperature was introduced at the top of the packed bed at the constant velocity of 55 cc./sec. Packing materials used were lead shot, ordinary sand, and crushed calcite. Diffusing materials used were benzene and water.

After the equipment had been immersed for 30 to 60 min. in the constant-temperature bath, air was introduced. Prior to the introduction of air, water or benzene diffuses upward and reaches the saturated state. This diffusion is expressed by the following basic equation:

$$\frac{\partial p}{\partial \theta} = \frac{De}{\epsilon}\frac{\partial^2 p}{\partial x^2} \tag{1}$$

$$\left.\begin{array}{llll} p = p_s & x = 0 & \theta = \theta \\ \partial p/\partial x = 0 & x = X & \theta = \theta \\ p = p_s & x = X & \theta = \infty \\ p = 0 & x = X & \theta = 0 \end{array}\right| \tag{2}$$

If one assumes that packing material is located above the liquid, the relationship between the concentration radient at the liquid surface and time would be

$$\left[\frac{\partial(p/p_s)}{\partial x}\right]_{x=0} = -\frac{2}{x}\left[e^{-(\pi/2)^2\tau} + e^{-9(\pi/2)^2\tau} + e^{-25(\pi/2)^2\tau} + \cdots\right] \tag{3}$$

where

$$\tau = De\theta/\epsilon X^2$$

Figure 2 illustrates the calculated results when $D_e$ = 0.03 sq. cm./sec., $X$ = 8, 12, and 16 cm., and $\epsilon$ = 0.5. It is shown also that 5, 11.5, and 20 min. are required to reach a saturated state at a liquid surface for three different bed depths.

The time necessary for reaching the saturated state after the introduction of air would be solved under the following conditions:

$$\left.\begin{array}{llll} p = p_s & x = x & \theta = 0 \\ p = p_s & x = 0 & \theta = \theta \\ p = p_s & x = X & \theta = 0 \\ p = 0 & x = X & \theta < 0 \end{array}\right| \tag{4}$$